WIRRAL

Mathematics

West Kirby Primary School

Progression in Calculation

This progression document is intended to support the teaching of written calculation strategies and is complimented by a sequence of CPD to secure subject knowledge.

The progression line reflects the expectations (set out within the appendix National Curriculum 2014)

It is intended to be a working document.

Progression in Addition

Addition is commutative.
Addition of positive numbers will give a larger answer than the start number as you are adding to the set.

Simpler Case → Crossing boundary → '0' as a place holder → Both involved → Mixed number of digits → More than 2 sets involved

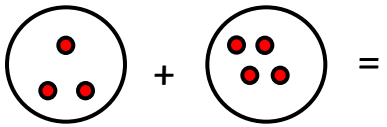
Context based experiences at each level of development - money, measures, real-life

Developmental

EY & Y1

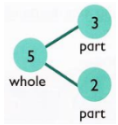
Use of objects, number tracks & number lines, counters and base ten.

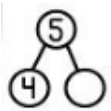
3 + 4 = 7



See progression – counting to calculating

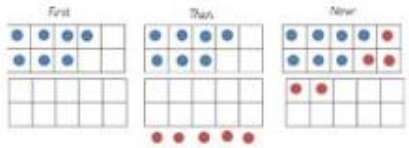
Part Whole Model





7	
	5

First, then, now



First there were _____.
Then _____ more were added.
Now there is _____.

Expanded

Y2 & Y3

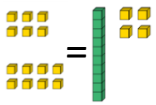
23 + 33 =

Tens	Ones
	
	
5	6

46 + 28 =

Tens	Ones
	
	
7	4

Ones - Regroup



Standard Algorithm

Years 3, 4, 5 & 6

```
  1 2 6
+ 4 3
-------
  1 6 9
```

```
    2 8
+   4 3
-------
  7 1
-------
  1
```

Worked examples

789 + 642 becomes

```
    7  8  9
+   6  4  2
-------------
  1  4  3  1
     1  1
```

Answer: 1431

- Understanding of place value places
- Add multiples of 10 (100) fluently
- Partition numbers into 100s 10s 1s and recombining
- Fluent in adding single digit numbers to 20

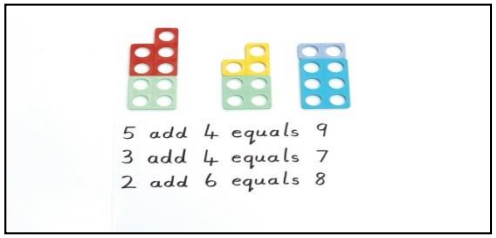
1s + 1s | 10s 1s + 1s | 10s 1s + 10s 1s | 100s 10s 1s + 1s | 100s 10s 1s + 10s 1s

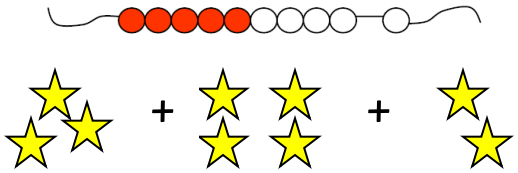
100s 10s 1s + 100s 10s 1s | 1000s 100s 10s 1s | 1s + 0.1s | 0.1s + 0.1s | 1s + 0.1s 0.01s | Mixed whole numbers & decimals

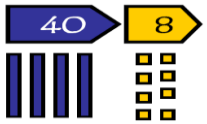
Misconceptions

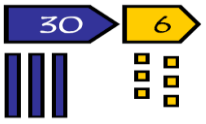
- Estimating first to see if their answer 'makes sense'
- Setting out when working in columns – confusion over the place value
- Confusion of 'teen' and 'ty'
- Number bonds not always quick rapid recall
- Confuse vocabulary more/less, add/subtract, difference between, 1's not units
- Starting from the left
- Decimal points not lined up
- Forgetting to add the digit that has been carried
- Missing numbers in calculations
  e.g. ___ + 71 = 95
- Balancing calculations
  e.g .21 + ___ = ___ + 12
  50
- Vocabulary – the word **Sum** is to be used only when adding.

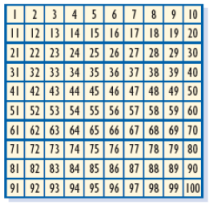
Models & Images

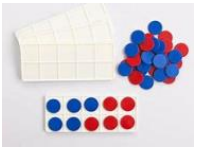


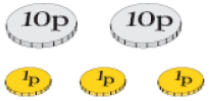


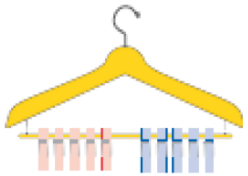






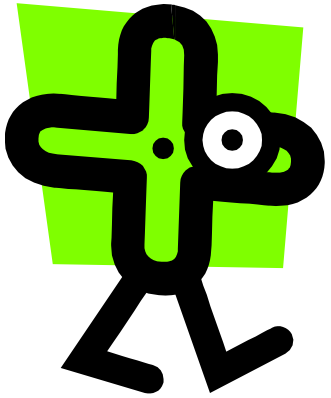






Linked Vocabulary

Add
More
Sum
Total
Equals
Is equal to
Greater
Plus
Addition
Increase
Subitise



Progression in Subtraction

- *Can be removal from set or finding the difference*
- *It is NOT commutative*

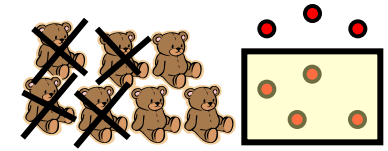
Simpler Case → Crossing boundary → '0' as a place holder → Both involved → Mixed number of digits → More than 2 sets involved

Context based experiences at each level of development - money, measures, real-life

Developmental

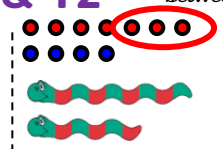
FS, Y1 & Y2

7 – 4 = *Use of objects, number tracks & number lines*



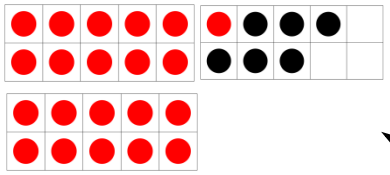
See progression – counting to calculating

What is the difference between 7 and 4

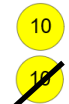


= 3

27 - 6 = 21

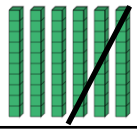


27 - 14 = 13

Tens	Ones
	
1	3

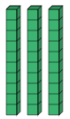
Y2

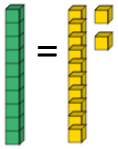
67 – 34 =

Tens	Ones
	
3	3

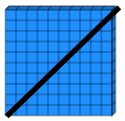
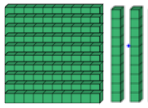
Y2 & Y3

32 - 15 = **Tens - regroup**

Tens	Ones
	
1	7



126 - 43 = **Hundreds- regroup**

Hundreds	Tens	Ones
		
	8	3

Standard Algorithm

Years 3, 4, 5 & 6

$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}6 \\ -\ 2\,8 \\ \hline 1\,8 \end{array}$$

$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}29 \\ -\ 1\,3\,2 \\ \hline 2\,9\,7 \end{array}$$

Worked Examples

932 – 457 becomes

$$\begin{array}{r} {}^{8}\not{9}\,{}^{12}\not{3}\,{}^{1}2 \\ -\ 4\,5\,7 \\ \hline 4\,7\,5 \end{array}$$

Removal from set, decomposition

- Subtract single digits confidently mentally
- Recall number bonds up to 20
- Partition numbers and recombine
- Understand place value and 're-group'
- Fluent in subtracting multiples of 10/100/1000

Finding the difference

- Subtract single digits confidently mentally
- Bridge to the nearest 10
- Add multiple to 10/100 to multiples / count fwds/bwds in 1-/100 from any given number
- Partition 100s 10s 1s
- Add several numbers mentally
- Secure addition strategy

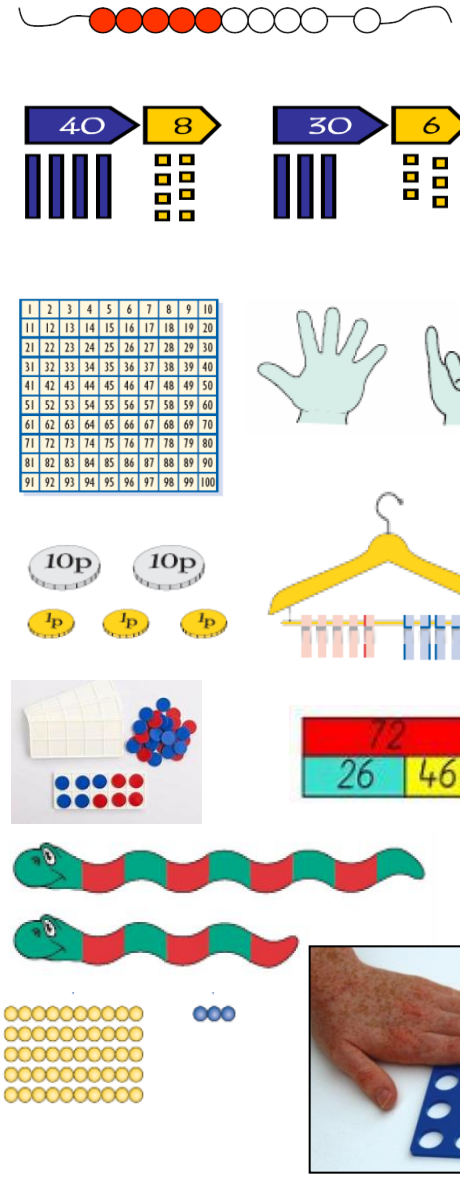
1s - 1s | 10s 1s - 1s | 10s 1s - 10s 1s | 100s 10s 1s - 1s | 100s 10s 1s - 10s 1s

100s 10s 1s - 100s 10s 1s | 1000s 100s 10s 1s | 1s - 0.1s | 0.1s - 0.1s | 1s - 0.1s 0.01s | Mixed whole numbers & decimals

Misconceptions

- Estimating first to see if their answer 'makes sense'
- Setting out when working in columns – confusion over the place value
- Confusion of 'teen' and 'ty'
- Using in number line – count start number so calculation is out by 1
- Misunderstanding regarding place value and concept of exchanging **10s** for 1s, **100s** for **10s** etc
- **Lack of understanding that when subtracting from a number that the answer will be smaller than start number as removing from it**
- **Children switch the digits around to be able to 'do' the calculation (believe it is commutative as with +/x)**
- Forget to carry (regroups) and or putting the carried number as part of the answer.
- Missing number problems e.g. __ - 24 = 45
- Choosing which method to use
- Decomposition with zeros e.g. 6004 – 1256=

Models & Images



Linked Vocabulary

Take
Take-away
Leave
Left
Fewer
Less than
Decrease
Difference between
Minus
Subtract
Subtraction
Equals / equal to

Progression in Multiplication (Short multiplication)

Multiplication is commutative.
Answer will be larger

Simpler Case (1x 10/100 – as in examples given) → Multiples of 10/100 (3x235) → '0' as a place holder → Both involved → Mixed number of digits

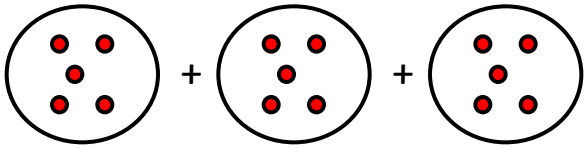
Context based experiences at each level of development - money, measures, real-life

Developmental

Y1 & 2

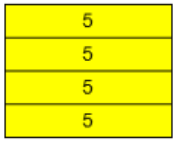
3 x 5 = 15

Use of objects, number tracks & number lines. Link initially to repeated +

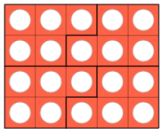


Representation as an array using a variety of apparatus (Dienes, pegs, counters etc)

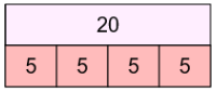
4 x 5 =  or 



Cuisenaire rods



Numicon



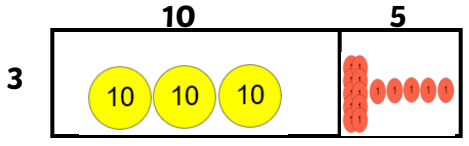
Bar Model

Expanded

Teach these side by side

Grid Method

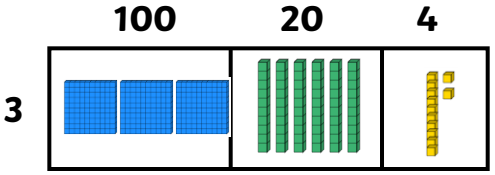
3 x 15 =

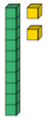


Re-group 

30 + 15 = **45**

3 x 124 =



Regroup 

= **372**

Standard Algorithm

Years 3, 4, 5 & 6

```
  15
x  3
----
  45
----
  1
```

```
  126
x   3
-----
  378
-----
  1
```

- Recall tables up to 12x12 (by the end Y4 within 6 seconds)
- Partition numbers into 100s 10s 1s
- Multiply by 10/100 / 1000
- Secure addition strategy for calculating total

NB. As children become secure in this method there may no longer be a need to write the calculations in brackets

1s x 1s | 1s x 10s 1s | 1s x 100s 10s 1s | 1s x 1000s 100s 10s 1s | 1s x 0.1s | 1s+0.1s 0.01s | Mixed whole numbers & decimals

Worked examples

2741 × 6 becomes

```
      2  7  4  1
   ×              6
-------------------
   1  6  4  4  6
      4  2
```

Answer: 16 446

Progression in Multiplication (Long multiplication)

Multiplication is commutative.

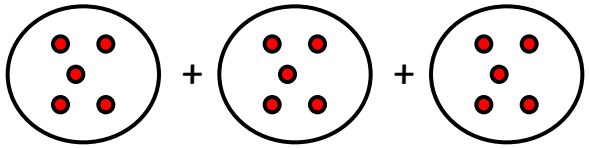
Simpler Case (1x 10/100 – as in examples given) → Multiples of 10/100 (3x235) → '0' as a place holder → Both involved → Mixed number of digits

Context based experiences at each level of development - money, measures, real-life

Developmental Y1 & 2

3 x 5 = 15

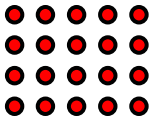
Use of objects, number tracks & number lines. Link initially to repeated +



Representation as an array using a variety of apparatus (Dienes, pegs, counters etc)

Times tables and associate division facts.

4 x 5 =



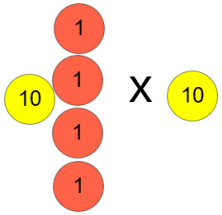
Practical work

- ***NB. use apparatus to model 10s 1s x10s 1s & 100s 10s 1s x 10s 1s etc.***

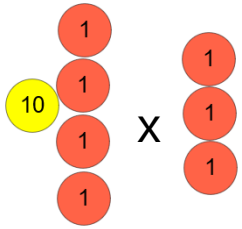
Expanded Y3 & 4

14 x 13 = 182

14 x 10 = 140



14 x 3 = 42



Standard Algorithm Y5 & 6

```
  14
x 13
----
  42
 1
 140
----
 182
```

```
  124
x  26
-----
  744
 1 2
 2480
-----
 3224
 1 1
```

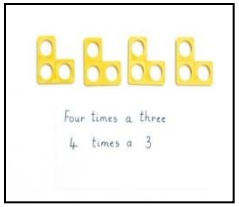
- Recall tables up to 12x12 (by the end Y4)
- Partition numbers into 100s 10s 1s
- Multiply by 10/100
- Secure addition strategy calculating total

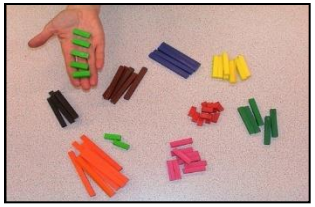
10s 1s x 10s 1s 100s 10s 1s x 10s 1s 1000s 100s 10s 1s x 10s 1s Decimals up to 2dp x whole numbers

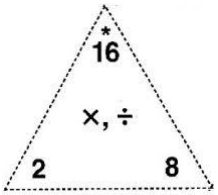
Misconceptions

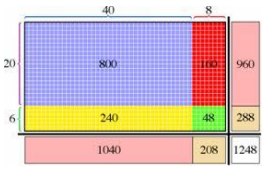
- Understanding on multiplying by 10/100 and what happens to place value of the number
- Rapid recall of multiplication tables is not secure and impacting of accuracy of calculation
- Interpretation of digits in the T/H columns as single digits eg 4x3 instead of 4x30
- Carrying units – either forgetting to carry or putting the number in the wrong column/place
- Place value, particularly with decimals.
- Long multiplication, using zero as a place holder
- Doubles & near doubles
- Multiplying by $\frac{1}{2}$

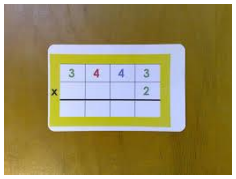
Models & Images











Linked Vocabulary

Repeated addition
Groups of
Lots of
Multiple
Multiply
Multiplication
Times
Product
Array

Progression in Division

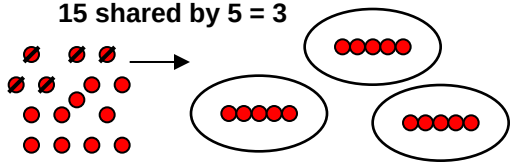
- *Division can be sharing or grouping*
- *Division is the inverse of multiplication*

Simpler Case → Within known x tables → Using multiples → Mixture of tens and ones → Within less familiar x tables

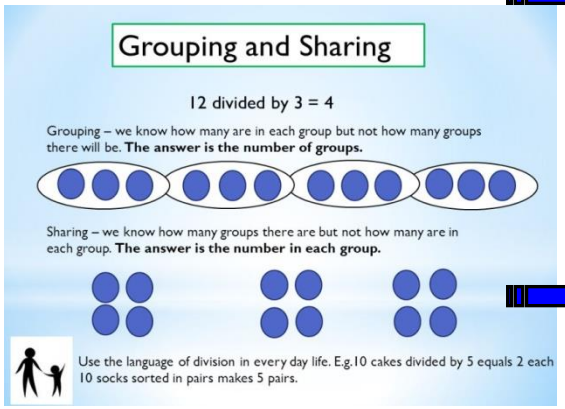
Context based experiences at each level of development - money, measures, real-life

Developmental Y1

15 shared by 5 = 3



Use of objects and number tracks . Link initially to repeated subtraction

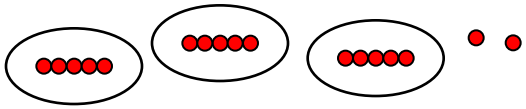


- Count in multiples of # from/back from 0
- Recall known facts for all multiplication tables up to 12x12 (NB multiplication squares can be used as a supporting model/image)
- Know related facts – doubling, x10
- Confidently subtract mentally

Developmental Y2

Times tables and associate division facts.

17 ÷ 3 = 5 r 2



24 ÷ 2 = 12

24 ÷ 12 = 2

2 x 12 = 24

12 x 2 = 24

25 ÷ 2 = 12 r 1

Year 3
- Remainders on a number line
- Bus stop (no remainders but carrying within)

Year 4
- Bus stop without carrying
- Bus stop with internal carrying
- Bus stop with remainders

Compact

Years 3, 4, 5 & 6

1 5
5 | 7 2 5

575 ÷ 5 =

1 1 5
5 | 5 7 2 5

Remainders should also be recorded as a decimal or fraction0

Standard Algorithm

'Stand alone' method → for efficiency

3's into 8 goes 2x with 2 remainder

2 8 r 1
3 | 8 2 5

3's into 25 go 8 remainder 1

12 into 1 won't go

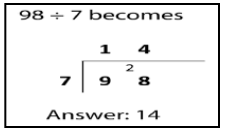
1 2 r4
12 | 1 1 4 2 8

12s in 14 is 1 remainder 2

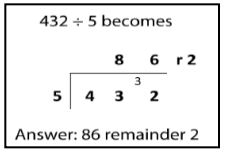
12 into 28 is 2 remainder 4

Worked examples

98 ÷ 7 becomes



432 ÷ 5 becomes



10s 1s ÷ 1s → 100s 10s 1s ÷ 1s → 1000s 100s 10s 1s ÷ 1s → 1s.0.1s ÷ 1s → 1s.0.01s ÷ 1s → 10s 1s. 0.1s ÷ 1s → 100s 10s 1s ÷ 10s 1s

Progression in Division (long division)

- *Division can be sharing or grouping*
- *Division is the inverse of multiplication*

Simpler Case → Within known/derived x tables → Any two-digit number

Context based experiences at each level of development - money, measures, real-life

Developmental

Secure division strategies for short division, recall and application of known & derived facts and making sensible approximations to support accuracy when calculating

Year 5 and Year 6 to be able to express a remainder as both a number, a decimal and fraction.

Compact

```
          1 5 r1
23 | 3 4 6
(1 x 23)   - 2 3
           -----
             1 1 6
(5 x 23)   - 1 1 5
           -----
             0 0 1
```

15 into 4 won't go

```
          2 8 r12
15 | 4 43 132
```

15 into 43 is 2 remainder 13

15's in 132 is …

Remainders should also be recorded as a decimal or fraction

At each stage children may need to make jottings to support the use of known and derived facts when calculating

Standard Algorithm

Year 6 only

```
          2 3 r10
33 | 7 6 9
   - 6 6
     -----
     1 0 9
   -   9 9
     -----
       1 0
```

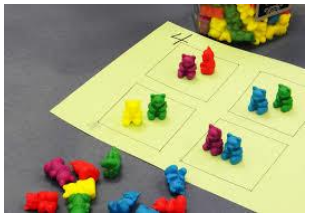
Children will need to be aware that the method for long division will not work with every calculation e.g. 24 | 2 3 7 Therefore they MUST have an alternative method they can use.

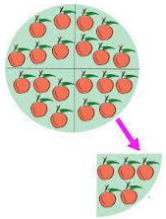
- Recall known facts for all multiplication tables up to 12x12
(NB multiplication squares can be used as a supporting model/image)
- Know related facts – doubling/halving, x10 x50 x25 x100
- Apply tables knowledge when approximating answers
- Confidently subtract mentally

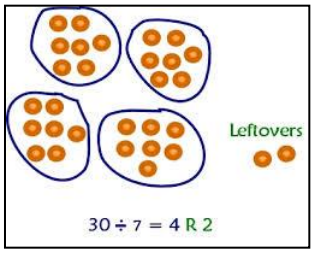
Misconceptions

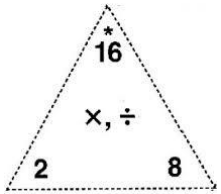
- Lack of understanding of 'remainders' and their importance to the context of the problem
- Insecure understanding of place value to know what each digit is representing
- Unable to derive facts from known facts and 'play' with numbers
- Approximations are wildly inaccurate so answers cannot be judged in the context of the problem/calculation
- No method to 'fall back' on where use of a formal method won't work
- Insecure with inverse operations
- When there is a remainder, is the answer rounded up (depending on the context)
- Halving and near halves.
- Starting in calculation in the wrong place

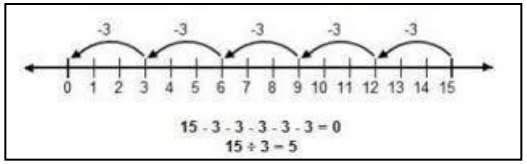
Models & Images

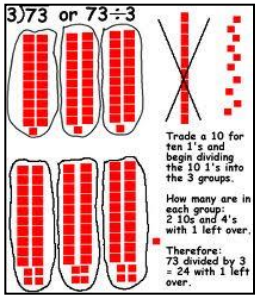






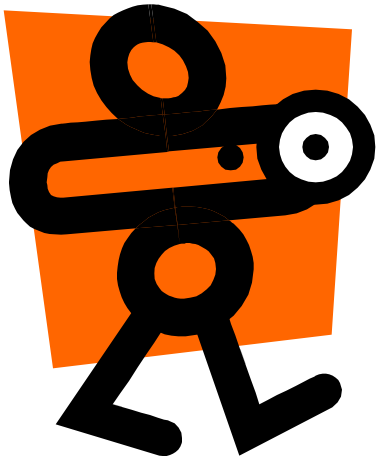






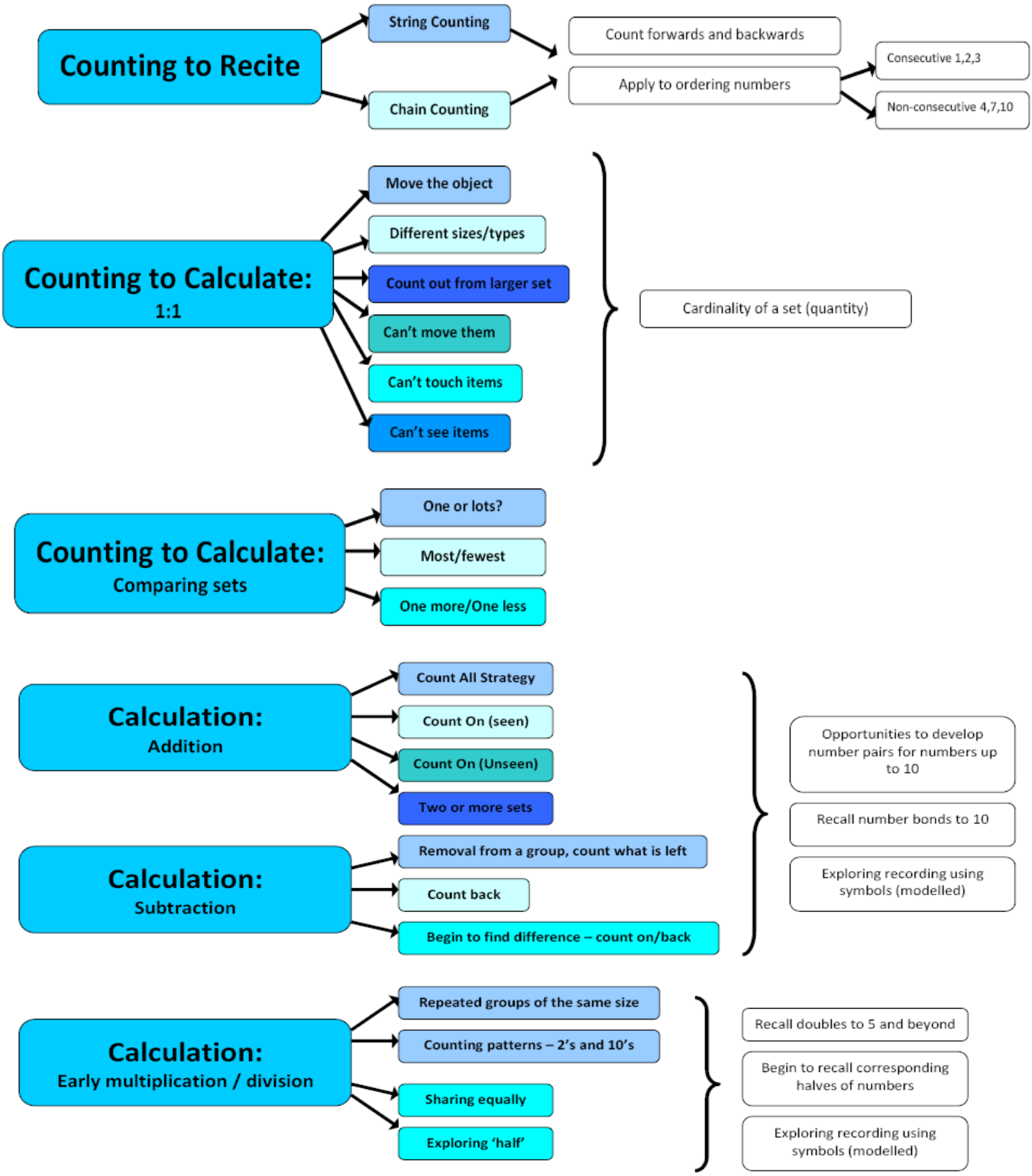
Linked Vocabulary

Divisor
Divisible
Divide
Group
Share
Chunk
Remainder
Sharing / shared
Equal groups



Supporting Materials

Counting into Calculating – A guide to progression



NB. Opportunities for children to meet these 5 core skills should be integrated. Children **DO NOT** have to be able to recite ALL numbers before they move to calculation. For example, if they are able to recite numbers to 3 then **they can** 1:1, compare, add and subtract up to 3.

National Curriculum 2014, Progression through the PoS: +/-

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Pupils should be taught to: read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs represent and use number bonds and related subtraction facts within 20 add and subtract one-digit and two-digit numbers to 20, including zero solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems	***Pupils should be taught to:*** • solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures applying their increasing knowledge of mental and written methods recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 • add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones a two-digit number and tens two two-digit numbers adding three one-digit numbers show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot • recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.	***Pupils should be taught to:*** • add and subtract numbers mentally, including: a three-digit number and ones a three-digit number and tens a three-digit number and hundreds • add and subtract numbers with up to three digits, using ***formal written methods*** of columnar addition and subtraction • estimate the answer to a calculation and use inverse operations to check answers • solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.	***Pupils should be taught to:*** • add and subtract numbers with up to 4 digits using the ***formal written methods*** of columnar addition and subtraction where appropriate • estimate and use inverse operations to check answers to a calculation • solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.	***Pupils should be taught to:*** • add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) • add and subtract numbers mentally with increasingly large numbers • use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.	***Pupils should be taught to:*** • use their knowledge of the order of operations to carry out calculations involving the four operations • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

National Curriculum 2014, Progression through the PoS: x/÷

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Pupils should be taught to:	***Pupils should be taught to:***	***Pupils should be taught to:***	***Pupils should be taught to:***	***Pupils should be taught to:***	***Pupils should be taught to:***
• solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays *with the support of the teacher*	• recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers • calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs • show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot • solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in context	• recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables • write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and ***progressing to formal written methods*** • solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.	• recall multiplication and division facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers • recognise and use factor pairs and commutativity in mental calculations • multiply two-digit and three-digit numbers by a one-digit number using ***formal written layout*** • solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.	• multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers • multiply and divide numbers mentally drawing upon known facts • divide numbers up to 4 digits by a one-digit number using the ***formal written method of short division*** and interpret remainders appropriately for the context • multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 • solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes • solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign • solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.	• multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • divide numbers up to 4 digits by a two-digit whole number ***using the formal written method of long division***, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context • divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context • use their knowledge of the order of operations to carry out calculations involving the four operations • solve problems involving addition, subtraction, multiplication and division • use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.